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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 986

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Unique Paper Code : 2352201102

Name of the Paper : DSC: Elements of Discrete Mathematics

Name of the Course : B.A. (Prog.)

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.
3. All questions are compulsory.
4. Marks are indicated.

P.T.O.

1. (a) Determine the following :

(i) Compute the truth table of the statement

$$(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p).$$

(ii) If $p \Rightarrow q$ is false, then determine the truth

value of $(\sim p) \vee (p \Leftrightarrow q)$. Explain your

answer. (7.5)

(b) Let $A = \mathbb{Z}$ (the set of integers). Define the

following relation R on A :

$a R b$ if and only if $|a - b| = 2$.

Determine whether the relation R on A is

reflexive, irreflexive, symmetric, asymmetric,

antisymmetric, or transitive. Is R an equivalence

relation on A ?

- (c) Prove by mathematical induction that if $A_1, A_2, \dots, A_n$ are any n sets, then

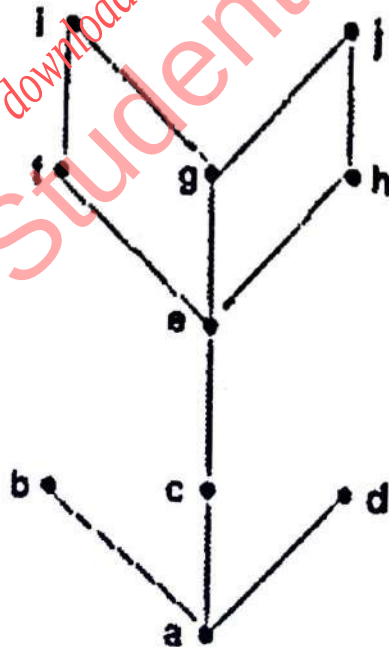
$$\overline{\bigcap_{i=1}^n A_i} = \bigcup_{i=1}^n \bar{A}_i \quad \text{where } \bar{A}_i \text{ denote the complement of the set } A_i. \quad (7.5)$$

2. (a) Let $X = \{1, 2, 3\}$. Consider the partial ordered set $(L, \leq)$ where $L = P(X)$ is the power set of X and $\leq$ is defined as, $U \leq V$ if and only if $U \subseteq V \forall U, V \in L$. Also consider partial ordered set S of all positive divisors of 30, with respect to the order that for any $a, b \in S$, $a \leq' b$ if and only if a divides b . Exhibit an order isomorphism between $(L, \leq)$ and $(S, \leq')$. Are the Hasse diagrams of two partial ordered sets $(L, \leq)$ and $(S, \leq')$ identical? (7.5)

P.T.O.

- (b) Let $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ of divisibility defined as $a \leq b$ if and only if a divides b . Draw a Hasse diagram for the subset $P = \{2, 3, 12, 18\}$ of $(\mathbb{N}_0, \leq)$. How many maximal and minimal elements are there in $(P, \leq)$? (7.5)

- (c) Find the lower and upper bounds along with greatest lower and least upper bound of the subsets $\{c, e\}$, $\{b, i\}$ in the following Hasse diagram. (7.5)



3. (a) Determine whether the relation $(\mathbb{Z}, \leq)$ on the set of all integers with the order "less than equal to" is a lattice. (7.5)

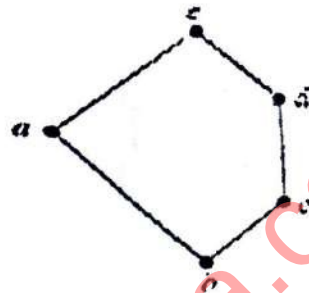
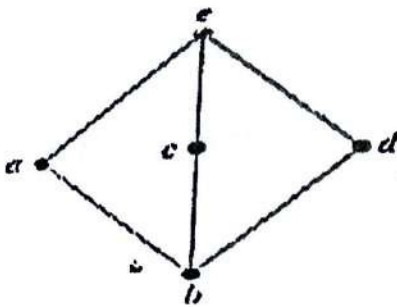
(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. Show that $m \leq n \Rightarrow l \wedge m \leq l \wedge n$ and $l \vee m \leq l \vee n$, for any $l, m, n \in L$. (7.5)

(c) Define a sublattice of a lattice L . Show that the interval $[x, y] = \{l \in L : x \leq l \leq y\}$, is a sublattice for any two elements $x, y \in L$ with $x \leq y$. (7.5)

4. (a) Define a distributive lattice. Prove that a homeomorphic image of a distributive lattice is distributive. (7.5)

P.T.O.

- (b) Does the following diamond and pentagonal lattices satisfy the distributive laws? (7.5)



- (c) Define a complemented lattice. Also show that $(P(M), \cap, \cup)$ is a complemented lattice for the power set $P(M)$ of a non-empty set M .

(7.5)

5. (a) What is Karnaugh map? Use Karnaugh map diagram to find a minimal form of the function

$$f(x, y, z, t) = x\bar{y} + xyz + \bar{x}\bar{y}\bar{z} + \bar{x}yz\bar{t}. \quad (7.5)$$

- (b) Find the DN form and CN form of the following Boolean functions

$$f(x, y, z) = x\bar{y} + x(\bar{y}z) + xyz \quad (7.5)$$

6. (a) Let $f(x, y, z) = xy\bar{z} + \bar{x}yz + \bar{x}y\bar{z}$. Find the implicants, prime implicants and essential prime implicants of $f(x, y, z)$.

$$\overline{(x(\bar{y}\bar{z}))} = \bar{x} + (y + z)(\bar{y} + \bar{z}). \quad (7.5)$$

- (b) Construct a logic circuit corresponding to Boolean function

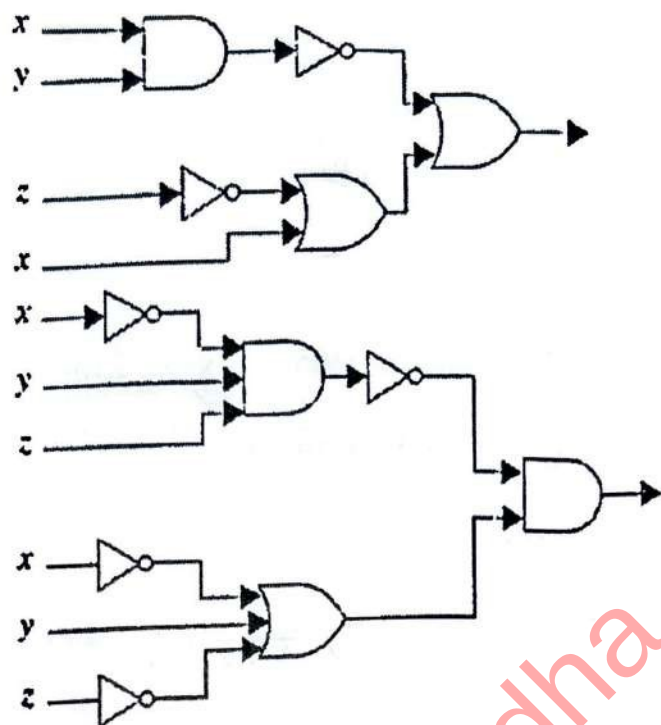
$$(i) f(x, y, z) = xyz' + yz' + x'y$$

$$(ii) f(x, y, z, w) = (x + y)(x' + z) + (z + w)' \quad (7.5)$$

- (c) Determine the output of each of these circuits

(7.5)

P.T.O.



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